NATIONAL ENGINEERING HANDBOOK

SECTION 4

HYDROLOGY

CHAPTER 16. HYDROGRAPHS

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NATIONAL ENGINEERING HANDBOOK

SECTION 4

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CHAPTER 16. HYDROGRAPHS

Purpose

Hydrographs, or some elements of them such as peak rates, are used in the planning and design of water control structures. They are also used to show the hydrologic effects of existing or proposed watershed projects.

Development of Hydrograph Relations

Runoff occurring on the uplands flows downstream in various patterns of flow which are affected by many factors such as spatial and temporal distribution of rainfall, rate of snowmelt, hydraulics of streams, watershed and channel storage, and others that are difficult to define. The graph of flow (rate versus time) at a stream section is the hydrograph, of which no two are exactly alike. There is no satisfactory mathematical analysis of flood hydrographs, and empirical relations have been developed, starting with the "Rational Method" in the 19th century, progressing to the Unit Hydrograph in the 1930's, and to more recent use of Dimensionless or Index Hydrographs. The empirical relations are simple elements from which as complex a hydrograph may be made as needed.

Present-day difficulties with hydrograph development lie in the precise estimation of runoff from rainfall (chapter 10) and determination of paths of flow (chapter 15).

Types of Hydrographs

This classification is a partial list, suitable for use in watershed work.

1. Natural hydrographs. Obtained directly from the flow records of a gaged stream.

2. Synthetic hydrographs. Obtained by using watershed parameters and storm characteristics to simulate a natural hydrograph.

3. Unit hydrograph. A natural or synthetic hydrograph for one inch of direct runoff. The runoff occurs uniformly over the watershed in a specified time.

4. Dimensionless hydrograph. Made to represent many unit hydrographs by using the time to peak and the peak rates as basic units and plotting the hydrographs in ratios of these units. Also called Index hydrograph.

Unit Hydrograph

In 1932, L.K. Sherman[1] advanced the theory of the unit hydrograph, or unit graph. The unit hydrograph procedure assumes that discharge at any time is proportional to the volume of runoff and that time factors affecting hydrograph shape are constant.

Both field data and laboratory tests have shown that the assumption of a linear relationship between watershed components is not strictly true. The non-linear relationships have not been investigated sufficiently to ascertain their effects on a synthetic hydrograph. Until more information is available the procedures of this chapter will be based on the unit hydrograph theory.

The fundamental principles of invariance and superposition make the unit graph an extremely flexible tool for developing synthetic hydrographs: 1) the hydrograph of surface runoff from a watershed due to a given pattern of rainfall is invariable, and 2) the hydrograph resulting from a given pattern of rainfall excess can be built up by superimposing the unit hydrograph due to the separate amounts of rainfall excess occurring in each unit period. This includes the principle of proportionality by which the ordinates of the hydrograph are proportional to the volume of rainfall excess.

The unit time or "unit hydrograph duration" is the optimum duration for occurrence of precipitation excess. In general, this unit time is approximately 20 percent of the time interval between the beginning of runoff from a short high-intensity storm and the peak discharge of the corresponding runoff.

The "storm duration" is the actual duration of the precipitation excess. The duration varies with actual storms. The dimensionless unit hydrograph used by SCS (figure 16.1) was developed by Victor Mockus. It was derived from a large number of natural unit hydrographs from watersheds varying widely in size and geographical locations. This dimensionless curvilinear hydrograph, also shown in table 16.1, has its ordinate values expressed in a dimensionless ratio q/q_p or Qa/Q and its abscissa values as t/T_p. This unit hydrograph has a point of inflection approximately 1.70 times the time-to-peak (T_p) and the time-to-peak 0.2 of the time-of-base (T_b).

[1]See References at end of chapter.

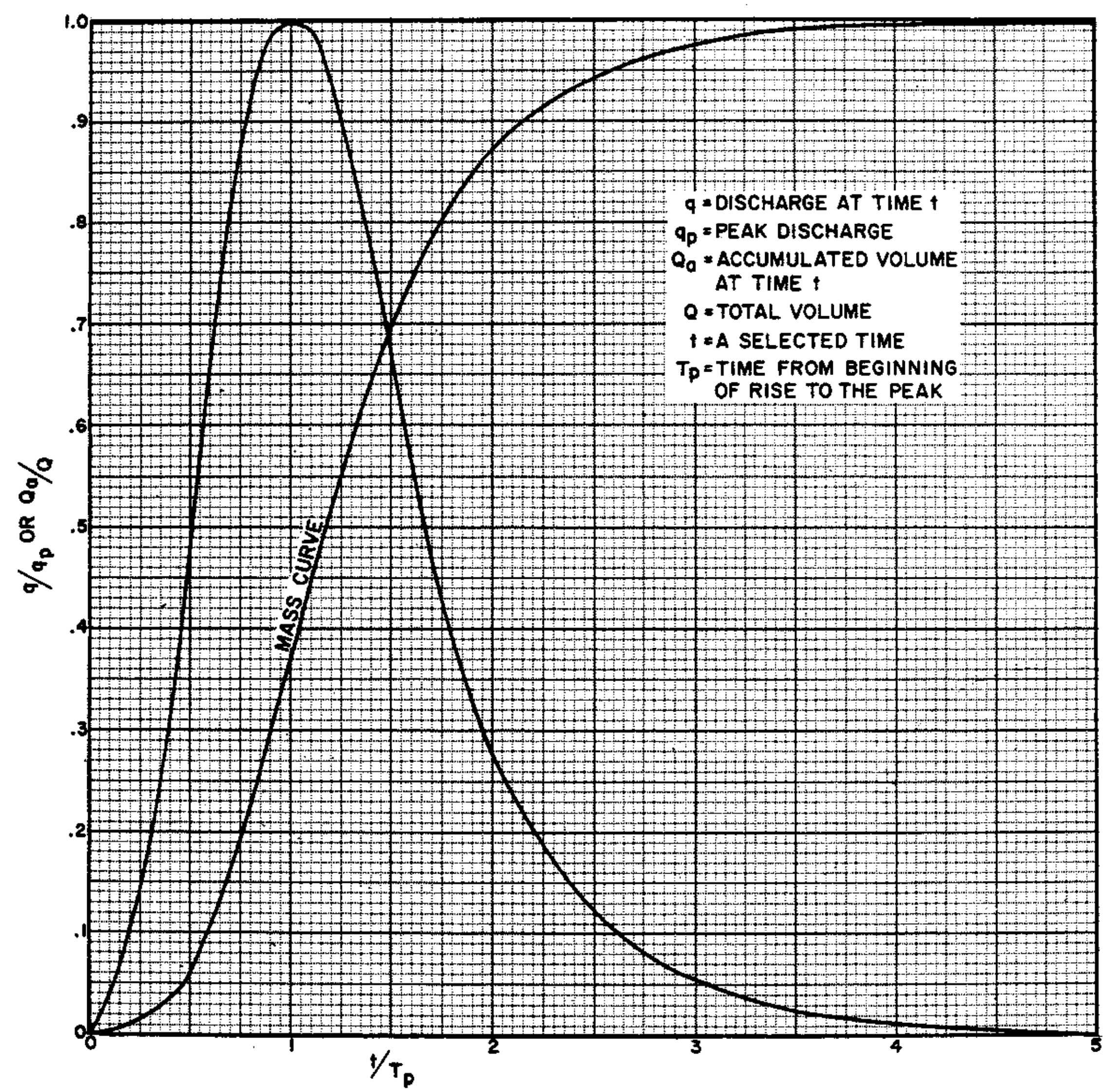

Figure 16.1 Dimensionless unit hydrograph and mass curve

Table 16.1 Ratios for dimensionless unit hydrograph and mass curve.

Time Ratios (t/T_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0	.000	.000
.1	.030	.001
.2	.100	.006
.3	.190	.017
.4	.310	.035
.5	.470	.065
.6	.660	.107
.7	.820	.163
.8	.930	.228
.9	.990	.300
1.0	1.000	.375
1.1	.990	.450
1.2	.930	.522
1.3	.860	.589
1.4	.780	.650
1.5	.680	.705
1.6	.560	.751
1.7	.460	.790
1.8	.390	.822
1.9	.330	.849
2.0	.280	.871
2.2	.207	.908
2.4	.147	.934
2.6	.107	.953
2.8	.077	.967
3.0	.055	.977
3.2	.040	.984
3.4	.029	.989
3.6	.021	.993
3.8	.015	.995
4.0	.011	.997
4.5	.005	.999
5.0	.000	1.000

<u>Elements of a Unit Hydrograph</u>
The dimensionless curvilinear unit hydrograph (figure 16.1) has 37.5% of the total volume in the rising side, which is represented by one unit of time and one unit of discharge. This dimensionless unit hydrograph also can be represented by an equivalent triangular hydrograph having the same units of time and discharge, thus having the same percent of volume in the rising side of the triangle (figure 16.2).

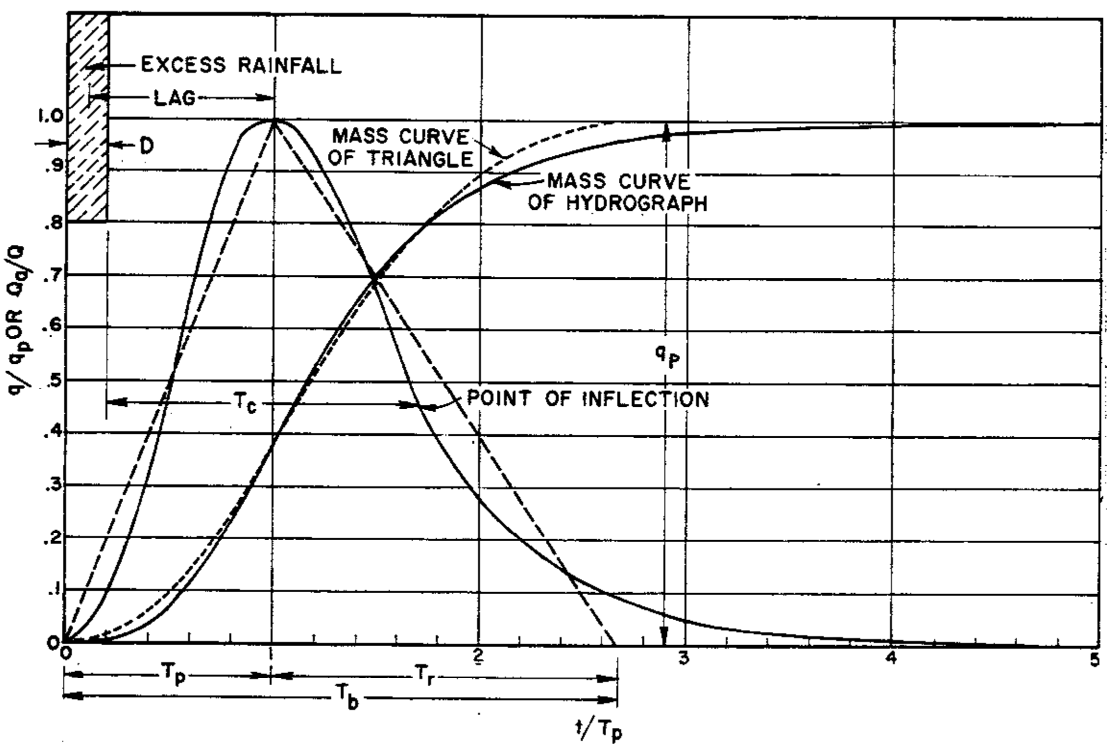

Figure 16.2 Dimensionless curvilinear unit hydrograph and equivalent triangular hydrograph

This allows the base of the triangle to be solved in relation to the time to peak using the geometry of triangles. Solving for the base length of the triangle, if one unit of time T_p equals .375 of volume:

$$T_b = \frac{1.00}{.375} = 2.67 \text{ units of time,}$$

$$T_r = T_b - T_p = 1.67 \text{ units of time or } 1.67\ T_p.$$

These relationships are useful in developing the peak rate equation for use with the dimensionless unit hydrograph.

Peak Rate Equation

From figure 16.2 the total volume under the triangular unit hydrograph is:

$$Q = \frac{q_p\ T_p}{2} + \frac{q_p\ T_r}{2} = \frac{q_p}{2}(T_p + T_r) \qquad \text{(Eq. 16.1)}$$

With Q in inches and T in hours, solve for peak rate q_p in inches per hour:

$$q_p = \frac{2Q}{T_p + T_r} \qquad \text{(Eq. 16.2)}$$

$$\text{Let} \quad K = \frac{2}{1 + \frac{T_r}{T_p}} \qquad \text{(Eq. 16.3)}$$

$$\text{Therefore} \quad q_p = \frac{KQ}{T_p} \qquad \text{(Eq. 16.4)}$$

In making the conversion from inches per hour to cubic feet per second and putting the equation in terms ordinarily used, including drainage area "A" in square miles, and time "T" in hours, equation 16.4 becomes the general equation:

$$q_p = \frac{645.33 \times K \times A \times Q}{T_p} \qquad \text{(Eq. 16.5)}$$

Where q_p is peak discharge in cubic feet per second (cfs) and the conversion factor 645.33 is the rate required to discharge one inch from one square mile in one hour.

The relationship of the triangular unit hydrograph, $T_r = 1.67\ T_p$, gives K = 0.75. Then substituting into equation 16.5 gives:

$$q_p = \frac{484\ A\ Q}{T_p} \qquad \text{(Eq. 16.6)}$$

Since the volume under the rising side of the triangular unit hydrograph is equal to the volume under the rising side of the curvilinear dimensionless unit hydrograph in figure 16.2, the constant 484, or peak rate factor, is valid for the dimensionless unit hydrograph in figure 16.1.

Any change in the dimensionless unit hydrograph reflecting a change in the percent of volume under the rising side would cause a corresponding change in the shape factor associated with the triangular hydrograph and therefore a change in the constant 484. This constant has been known to vary from about 600 in steep terrain to 300 in very flat swampy country. The E&WP Unit hydrologist should concur in the use of a dimensionless unit hydrograph other than figure 16.2. If for some reason it becomes necessary to vary the dimensionless shape of the hydrograph to perform a special job, the ratio of the percent of total volume in the rising side of the unit hydrograph to the rising side of a triangle is a useful tool in arriving at the peak rate factor.

Figure 16.2 shows that:

$$T_p = \frac{\Delta D}{2} + L \qquad \text{(Eq. 16.7)}$$

where ΔD is the duration of unit excess rainfall and L is the watershed lag in hours. The lag (L) of a watershed is defined (chapter 15) as the time from the center of mass of excess rainfall (ΔD) to the time to peak (T_p) of a unit hydrograph. From equation 16.6:

$$q_p = \frac{484\ A\ Q}{\frac{\Delta D}{2} + L} \qquad \text{(Eq. 16.8)}$$

The average relationship of lag (L) to time of concentration (T_c) is $L = 0.6\ T_c$ (chapter 15).

Substituting in equation 16.8, the peak rate equation becomes:

$$q_p = \frac{484\ A\ Q}{\frac{\Delta D}{2} + 0.6\ T_c} \qquad \text{(Eq. 16.9)}$$

The time of concentration is defined in two ways in chapter 15: 1) the time for runoff to travel from the furthermost point in the watershed to one point in question, and 2) the time from the end of excess rainfall to the point of inflection of the unit hydrograph.

These two relationships are important since T_c is computed under the first definition and ΔD, the unit storm duration, is used to compute the time to peak (T_p) of the unit hydrograph. This in turn is applied to all of the points on the abscissa of the dimensionless unit hydrograph using the ratio t/T_p as shown in table 16.1.

The dimensionless unit hydrograph shown in figure 16.2 has a time to peak at one unit of time and point of inflection at approximately 1.7 units of time. Using the relationships Lag = 0.6 T_c and the point of inflection = 1.7 T_p, ΔD will be .2 T_p. A small variation in ΔD is permissible, however, it should be no greater than .25 T_p. See example 1.

Using the relationship shown on the dimensionless unit hydrograph, figure 16.2 to compute the relationship of ΔD to T_c:

$$T_c + \Delta D = 1.7\ T_p \qquad \text{(Eq. 16.10)}$$

$$\frac{\Delta D}{2} + .6\ T_c = T_p \qquad \text{(Eq. 16.11)}$$

Solving these two equations:

$$T_c + \Delta D = 1.7\ \left(\frac{\Delta D}{2} + .6\ T_c\right)$$

$$.15\ \Delta D = .02\ T_c$$

$$\Delta D = .133\ T_c \qquad \text{(Eq. 16.12)}$$

Application of Unit Hydrograph

The unit hydrograph can be constructed for any location on a uniformly shaped watershed, once the values of q_p and T_p are defined (figure 16.3, areas A and B).

Area C in figure 16.3 is an irregularly shaped watershed having two uniformly shaped areas (C2 and C1) with a big difference in their time of concentration. This watershed requires the development of two unit hydrographs which may be added together forming one irregularly shaped unit hydrograph. This irregularly shaped unit hydrograph may be used to develop a flood hydrograph in the same way as the unit hydrograph developed from the dimensionless form (figure 16.1) is used to develop the flood hydrograph. See example 1 for area shown in figure 16.3. Also, each of the two unit hydrographs developed for areas C2 and C1 in figure 16.3 may be used to develop a flood hydrograph for its respective C2 and C1 areas. The flood hydrographs from each area are then combined to form the hydrograph at the outlet of area C.

There are many variables integrated into the shape of a unit hydrograph. Since a dimensionless unit hydrograph is used and the only parameters readily available from field data are drainage area and time of concentration, consideration should be given to dividing the watershed into hydrologic units of uniformly shaped areas. These divisions, if at all possible, should be no greater than 20 square miles in area and should have a homogeneous drainage pattern.

The "storm duration" is the actual time duration of precipitation excess. This time duration varies with actual storms and should not be confused with the unit time or unit hydrograph duration.

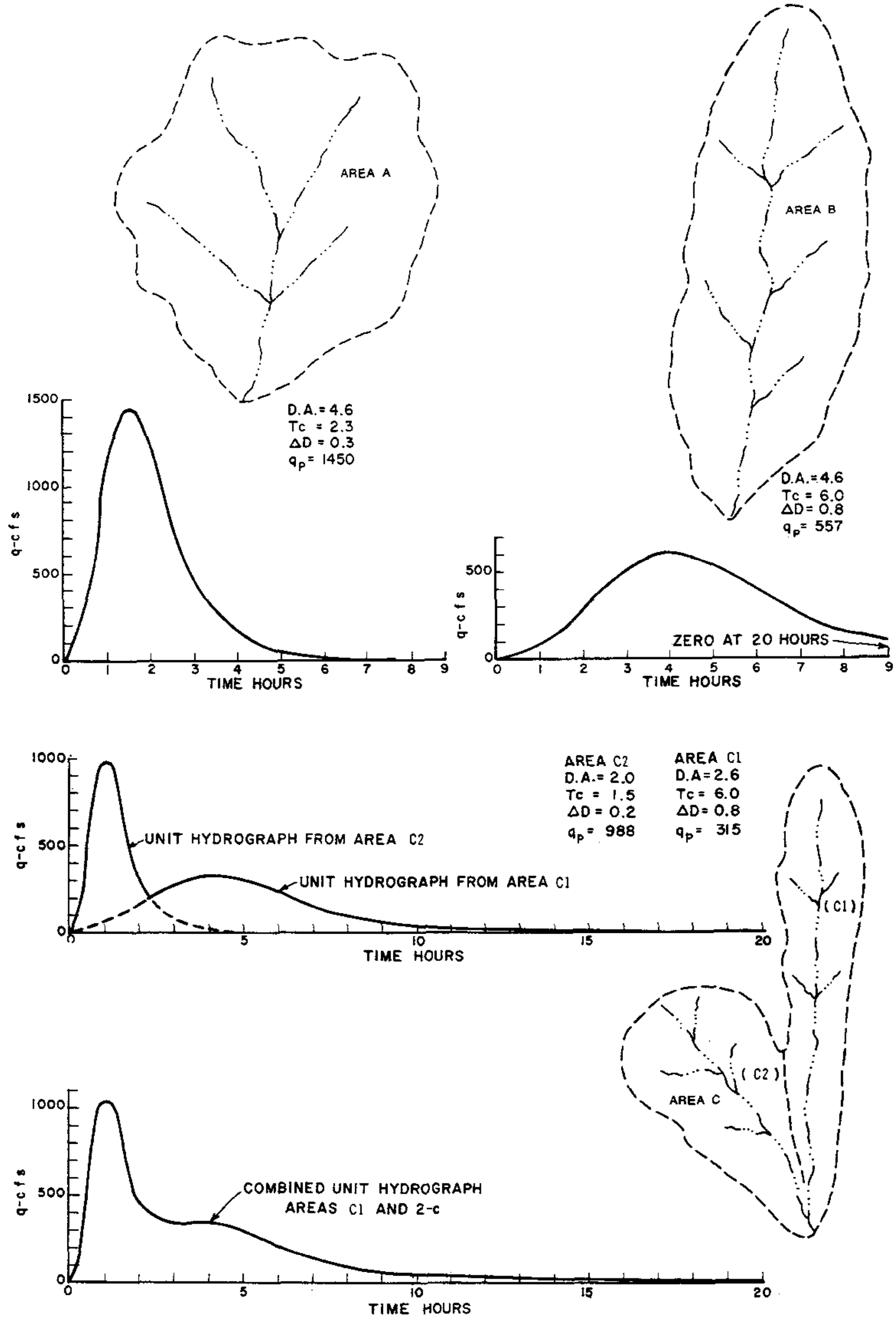

Figure 16.3 The effect of watershed shape on the peaks of unit hydrographs

Example 1

Develop a composite flood hydrograph using the runoff produced by the rainfall taken from a recording rain gage (figure 16.4(a)) on watershed (Area A) shown on figure 16.3.

Given the following information:

Drainage Area - 4.6 square miles

Time of Concentration - 2.3 hours

CN-85

Moisture Condition II

Storm Duration - 6 hours

Step 1. Develop and plot unit hydrograph.

Using equation 16.12, compute ΔD:

$$\Delta D = .133 \times 2.3 = .306 \quad \text{use } .30 \text{ hours}$$

Using equation 16.7, compute T_p:

$$T_p = \frac{.30}{2} + (.6 \times 2.3) = 1.53 \text{ hours}$$

Using equation 16.6, compute q for volume of runoff equal to one inch:

$$q_p = \frac{484 \times 4.6 \times 1}{1.53} = 1450 \text{ cfs}$$

The coordinates of the curvilinear unit hydrograph are shown in table 16.2 and the plotted hydrograph on figure 16.5.

Step 2. Tabulate the ordinates of the unit hydrograph from figure 16.5 in 0.3 hour increments (table 16.3a, column 2).

Step 3. Check the volume under unit hydrograph by summing the ordinates (table 16.3a, column 2) and multiplying by ΔD:

$$9898 \times 0.3 = 2969.4 \text{ cfs-hours}$$

Compare this figure with computed volume under unit hydrograph:

$$645.33 \times 4.6 = 2968 \text{ cfs-hours}$$

If these fail to check, re-read the coordinates from figure 16.5 and adjust if necessary until a reasonable balance in volume is attained.

Step 4. Tabulate the accumulated rainfall in .3 hour increments (table 16.4, column 2).

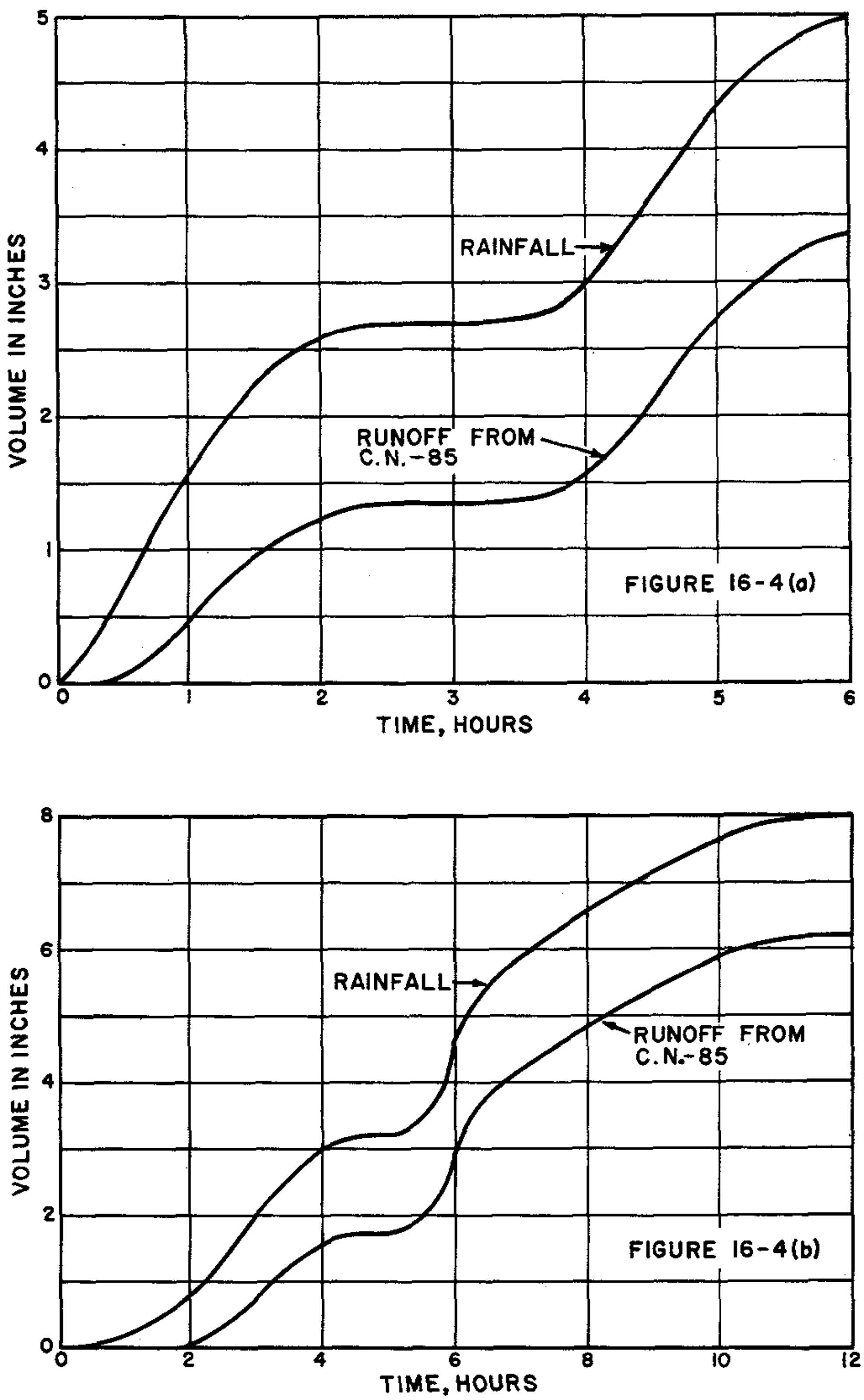

Figure 16.4 Accumulated rainfall and runoff for CN-85 taken from a recording rain gage.

Table 16.2. Computation of coordinates for unit hydrograph for use in Example 1.

1	2	3	4
Time Ratios (table 16.1)	Time (col 1 x 1.53)	Discharge Ratios (table 16.1)	Discharges (col 3 x 1450)
(t/T_p)	(hours)	(q/q_p)	(cfs)
.0	0	0	0
.1	.15	.030	44
.2	.31	.100	145
.3	.46	.190	276
.4	.61	.310	450
.5	.76	.470	682
.6	.92	.660	957
.7	1.07	.820	1189
.8	1.22	.930	1349
.9	1.38	.990	1435
1.0	1.53	1.000	1450
1.1	1.68	.990	1435
1.2	1.84	.930	1349
1.3	1.99	.860	1247
1.4	2.14	.780	1131
1.5	2.29	.680	986
1.6	2.45	.560	812
1.7	2.60	.460	667
1.8	2.75	.390	565
1.9	2.91	.330	479
2.0	3.06	.280	406
2.2	3.37	.207	300
2.4	3.67	.147	213
2.6	3.98	.107	155
2.8	4.28	.077	112
3.0	4.59	.055	80
3.2	4.90	.040	58
3.4	5.20	.029	42
3.6	5.51	.021	30
3.8	5.81	.015	22
4.0	6.12	.011	16
4.5	6.89	.005	7
5.0	7.65	0	0

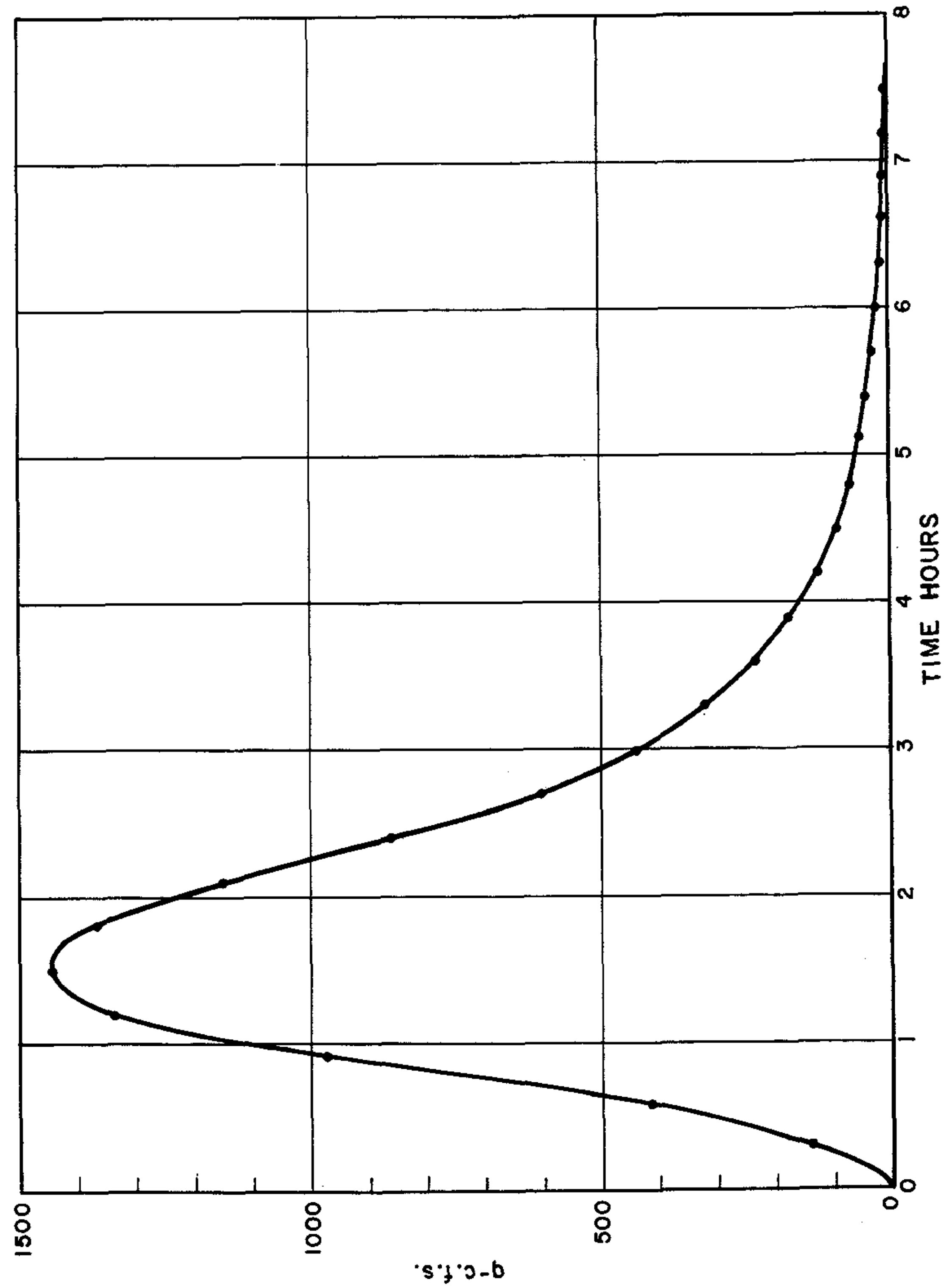

Figure 16.5 Unit hydrograph from example 1

Table 16.3. Computation of a flood hydrograph (example 1).

Table 16.3 (a)

(1) Time		(2) Unit Hyd.	(3) Flood Hyd.
	.09		
	.19		
	.24		
	.31		
	.42		
	.36		
	.25		
	.11		
	.05		
	.01		
	.0		
	.0		
	.06		
	.12		
	.18		
	.26		
	.33		
0	.27	0	0
.3	.12	140	0
.6	.0 →	420	17
.9		960	
1.2		1330	
1.5		1450	
1.8		1370	
2.1		1140	
2.4		860	
2.7		610	
3.0		440	
3.3		320	
3.6		230	
3.9		170	
4.2		120	
4.5		85	
4.8		70	
5.1		55	
5.4		40	
5.7		30	
6.0		20	
6.3		15	
6.6		10	
6.9		7	
7.2		4	
7.5		2	
7.8		0	
8.1			
8.4			
8.7			
9.0			
9.3			
9.6			
9.9			
10.2			
10.5			
10.8			
11.1			
11.4			
11.7			
12.0			
12.3			
12.6			
12.9			
13.2			
13.5			
13.7			

Total 9898

Table 16.3 (b)

(1) Time		(2) Unit Hyd.	(3) Flood Hyd.
	.09		
	.19		
	.24		
	.31		
	.42		
	.36		
	.25		
	.11		
	.05		
	.01		
	.0		
	.0		
	.06		
	.12		
	.18		
	.26		
0	.33	0	0
.3	.27	140	0
.6	.12	420	17
.9	.0 →	960	88
1.2		1330	
1.5		1450	
1.8		1370	
2.1		1140	
2.4		860	
2.7		610	
3.0		440	
3.3		320	
3.6		230	
3.9		170	
4.2		120	
4.5		85	
4.8		70	
5.1		55	
5.4		40	
5.7		30	
6.0		20	
6.3		15	
6.6		10	
6.9		7	
7.2		4	
7.5		2	
7.8		0	
8.1			
8.4			
8.7			
9.0			
9.3			
9.6			
9.9			
10.2			
10.5			
10.8			
11.1			
11.4			
11.7			
12.0			
12.3			
12.6			
12.9			
13.2			
13.5			
13.7			

Table 16.3 (c)

(1) Time		(2) Unit Hyd.	(3) Flood Hyd.
0		0	0
.3		140	0
.6	.09	420	17
.9	.19	960	88
1.2	.24	1330	275
1.5	.31	1450	594
1.8	.42	1370	984
2.1	.36	1140	1337
2.4	.25	860	1563
2.7	.11	610	1620
3.0	.05	440	1516
3.3	.01	320	1300
3.6	.0	230	1050
3.9	.0	170	838
4.2	.06	120	726
4.5	.12	85	765
4.8	.18	70	988
5.1	.26	55	1359
5.4	.33	40	1787
5.7	.27	30	2143
6.0	.12	20	2342
6.3	.0 →	15	2350
6.6		10	
6.9		7	
7.2		4	
7.5		2	
7.8		0	
8.1			
8.4			
8.7			
9.0			
9.3			
9.6			
9.9			
10.2			
10.5			
10.8			
11.1			
11.4			
11.7			
12.0			
12.3			
12.6			
12.9			
13.2			
13.5			
13.7			

Table 16.3 (d)

(1) Time		(2) Unit Hyd.	(3) Flood Hyd.
0		0	0
.3		140	0
.6		420	17
.9		960	88
1.2		1330	275
1.5		1450	594
1.8		1370	984
2.1		1140	1337
2.4		860	1563
2.7		610	1620
3.0		440	1516
3.3		320	1300
3.6		230	1050
3.9		170	838
4.2		120	726
4.5		85	765
4.8		70	988
5.1		55	1359
5.4		40	1787
5.7		30	2143
6.0		20	2342
6.3		15	2350
6.6		10	2170
6.9		7	1854
7.2		4	1488
7.5		2	1138
7.8	.09	0	840
8.1	.19		608
8.4	.24		438
8.7	.31		318
9.0	.42		233
9.3	.36		172
9.6	.25		128
9.9	.11		96
10.2	.05		72
10.5	.01		53
10.8	.0		38
11.1	.0		27
11.4	.06		18
11.7	.12		12
12.0	.18		7
12.3	.26		4
12.6	.33		2
12.9	.27		1
13.2	.12		0
13.5	.0 →		
13.7			

Total 33359

Table 16.4 Rainfall tabulated in 0.3 hour increments from plot of Rain Gage Chart, Figure 16.4a

1 Time	2 Accum. Rainfall	3 Accum.[1] Runoff	4 Incremental Runoff	5 Reversed Incremental Runoff
0	0			
.3	.37	.00	00	.09
.6	.87	.12	.12	.19
.9	1.40	.39	.27	.24
1.2	1.89	.72	.33	.31
1.5	2.24	.98	.26	.42
1.8	2.48	1.16	.18	.36
2.1	2.63	1.28	.12	.25
2.4	2.70	1.34	.06	.11
2.7	2.70	1.34	.00	.05
3.0	2.70	1.34	.00	.00
3.3	2.71	1.35	.01	.00
3.6	2.77	1.40	.05	.00
3.9	2.91	1.51	.11	.06
4.2	3.20	1.76	.25	.12
4.5	3.62	2.12	.36	.18
4.8	4.08	2.54	.42	.26
5.1	4.43	2.85	.31	.33
5.4	4.70	3.09	.24	.27
5.7	4.90	3.28	.19	.12
6.0	5.00	3.37	.09	.00

[1]Runoff computed using CN 85 moisture condition II.

Step 5. Compute the accumulated runoff (table 16.4, column 3) using CN-85, condition II.

Step 6. Tabulate the incremental runoff (table 16.4, column 4).

Step 7. Tabulate the incremental runoff <u>in reverse order</u> (table 16.4, column 5) and/or tabulate it on a strip of paper having the same line spacing as the paper used in step 2.

Step 8. Place the strip of paper between column 1 and column 2 of table 16.3(a) and slide down until the first increment of runoff (0.12) on the strip of paper is opposite the first discharge (140) on the unit hydrograph (column 2). Multiplying 0.12 x 140 = 16.8 (round to 17). Tabulate in column 3 opposite the arrow on the strip of paper.

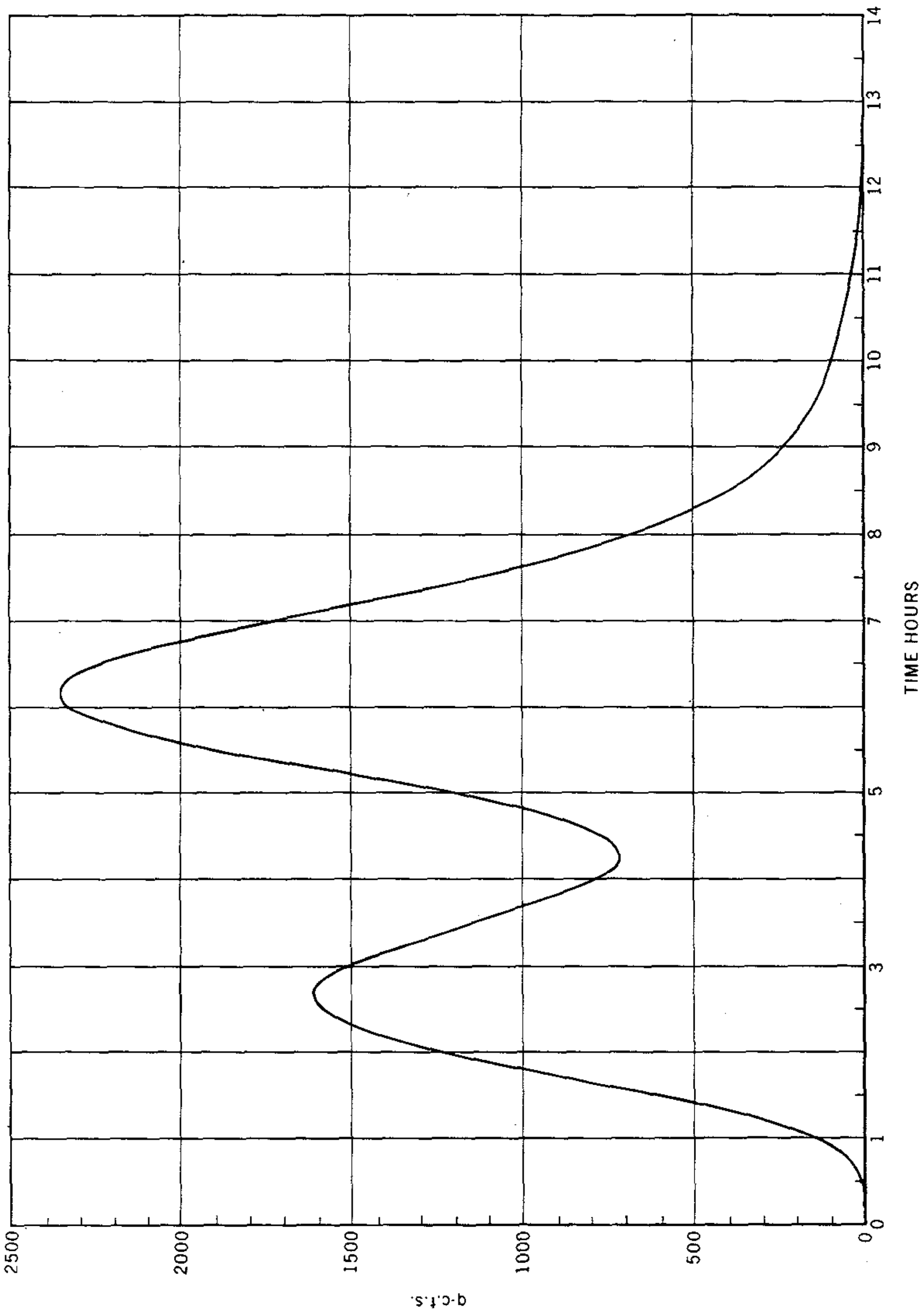

Figure 16.6 Composite flood hydrograph from example 1

Step 9. Move the strip of paper down one line (table 16.3(b)) and compute (0.12 x 420) + (.27 x 140) = 88.2 (round to 88). Tabulate in column 3 opposite the arrow on the strip of paper.

Continue moving the strip of paper containing the runoff down one line at a time and accumulatively multiply each runoff increment by the unit hydrograph discharge opposite the increment.

Table 16.3(c) shows the position of the strip of paper containing the runoff when the peak discharge of the flood hydrograph (2350 cfs) is reached. If only the peak discharge of the flood hydrograph is desired, it can be found by making only a few computations, placing the larger increments of runoff near the peak discharge of the unit hydrograph.

Figure 16.3(d) shows the position of the strip of paper containing the runoff at the completion of the flood hydrograph. The complete flood hydrograph is shown in column 3. These discharges are plotted at their proper time sequence on figure 16.6 which is the complete flood hydrograph for example 1.

Step 10. Check the volume under the flood hydrograph by summing the ordinates (table 16.3(d), column 3) and multiplying by ΔD. 33359 x .3 = 10007.7 cfs-hours, compared to computed volume, 645.33 x 4.6 x 3.37 = 10003.9 cfs-hours.

Example 2

Using the same data given in example 1, graphically develop a composite flood hydrograph using a triangle for the unit hydrograph.

Step 1. Plot the triangular unit hydrograph (dashed line) on figure 16.7: T_p = 1.53 hours, T_b = 4.08 hours.

Step 2. Compute the peak discharge for the first incremental triangular hydrograph by multiplying the first increment of runoff shown in table 16.4, column 4, by the peak discharge for one inch of runoff (1450). The peak of the first incremental triangular hydrograph is 1450 x .12 = 174. Since the storm did not produce runoff for the first increment of time and the zero point of the first incremental triangular hydrograph is plotted at 0.3 hours. The peak discharge of 174 cfs is plotted at 1.83 hours and end of the base is 4.38 hours. Continue developing and plotting incremental triangular hydrographs for each increment of runoff shown in table 16.4, column 4. Each incremental hydrograph is plotted one ΔD (0.3) hour later in time.

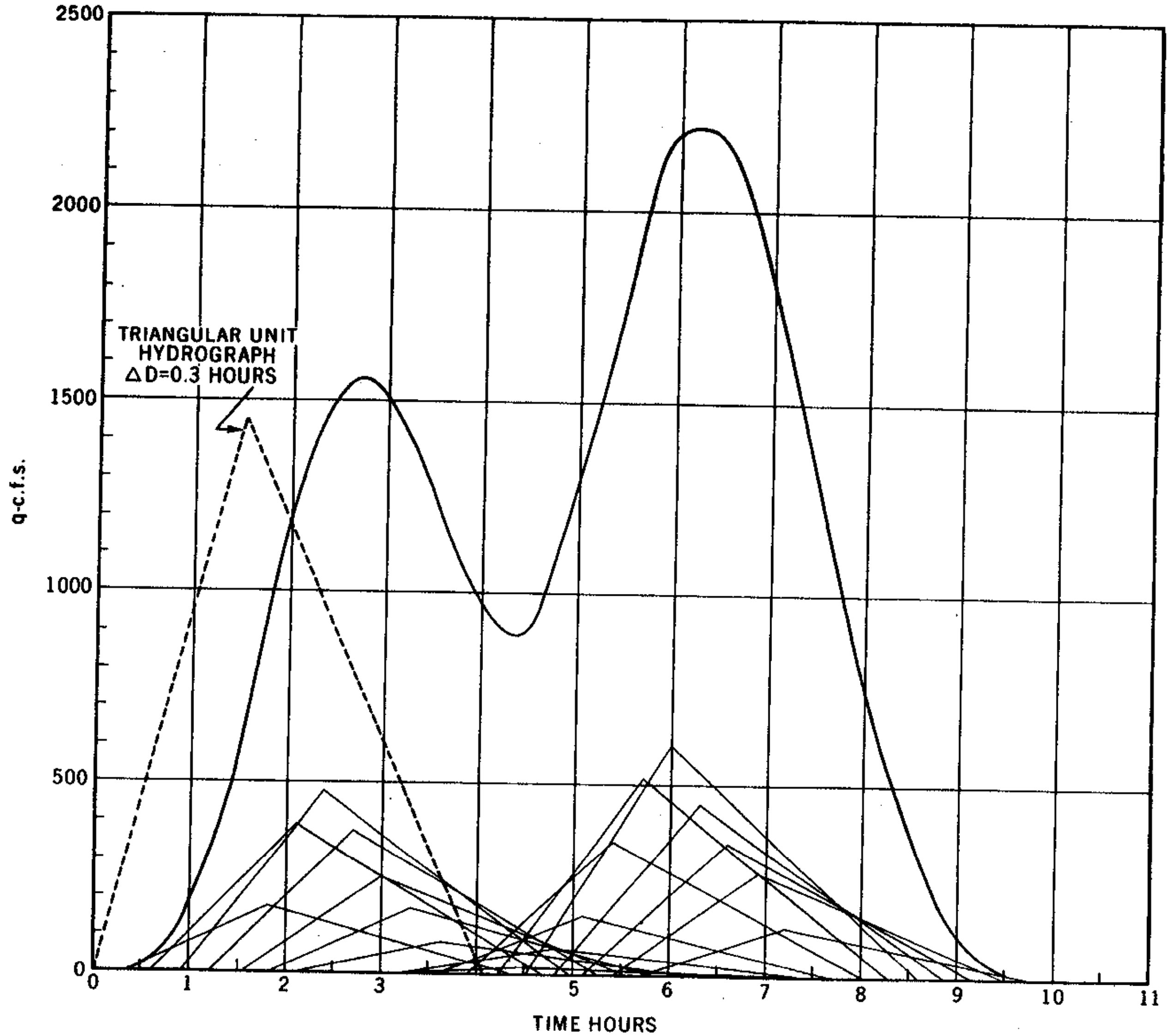

Figure 16.7 Composite flood hydrograph from example 2

Step 3. Sum the ordinates of each incremental triangular hydrograph at enough locations to make it possible to draw the completed flood hydrograph (figure 16.7). The composite peak is 2230 cfs.

Step 4. Check the area under the completed hydrograph and convert to cfs/hours, which is 40 sq. inches x 250 cfs-hours/sq. inch = 10,000 cfs-hours compared to the computed volume 645.33 x 4.6 x 3.37 = 10,003.9 cfs-hours. (Note: figure 16.7 has been reduced.)

Example 3

Using the same data given in example 1, but using a ΔD of 1.5 hours rather than 0.3 hour, graphically develop a composite flood hydrograph produced by the runoff from the rainfall shown on figure 16.4(a) and tabulated in table 16.5, column 4. This example will illustrate the effect of using a ΔD which is too large.

$$T_p = \frac{1.5}{2} + (.6 \times 2.3) = 2.13 \text{ hours}$$

$$T_b = 2.13 \times 2.67 = 5.68 \text{ hours}$$

$$q_p = \frac{484 \times 4.6 \times 1}{2.13} = 1043 \text{ cfs}$$

Following the same procedure outlined in example 2 of computing, plotting, and summing the ordinates of the incremental triangular hydrographs, a composite flood hydrograph is developed as shown in figure 16.8.

Table 16.5 Rainfall tabulated in 1.5 hour increments from plot of Rain Gage Chart, Figure 16.4a

Time	Accum. Rainfall	Accum. Runoff[1]	Incremental Runoff
0	0		
1.5	2.24	.98	.98
3.0	2.70	1.34	.36
4.5	3.62	2.12	.78
6.0	5.00	3.37	1.25

[1]Runoff computed using CN-85 moisture condition II.

The area under the composite flood hydrograph should be determined and the volume checked against the computed volume.

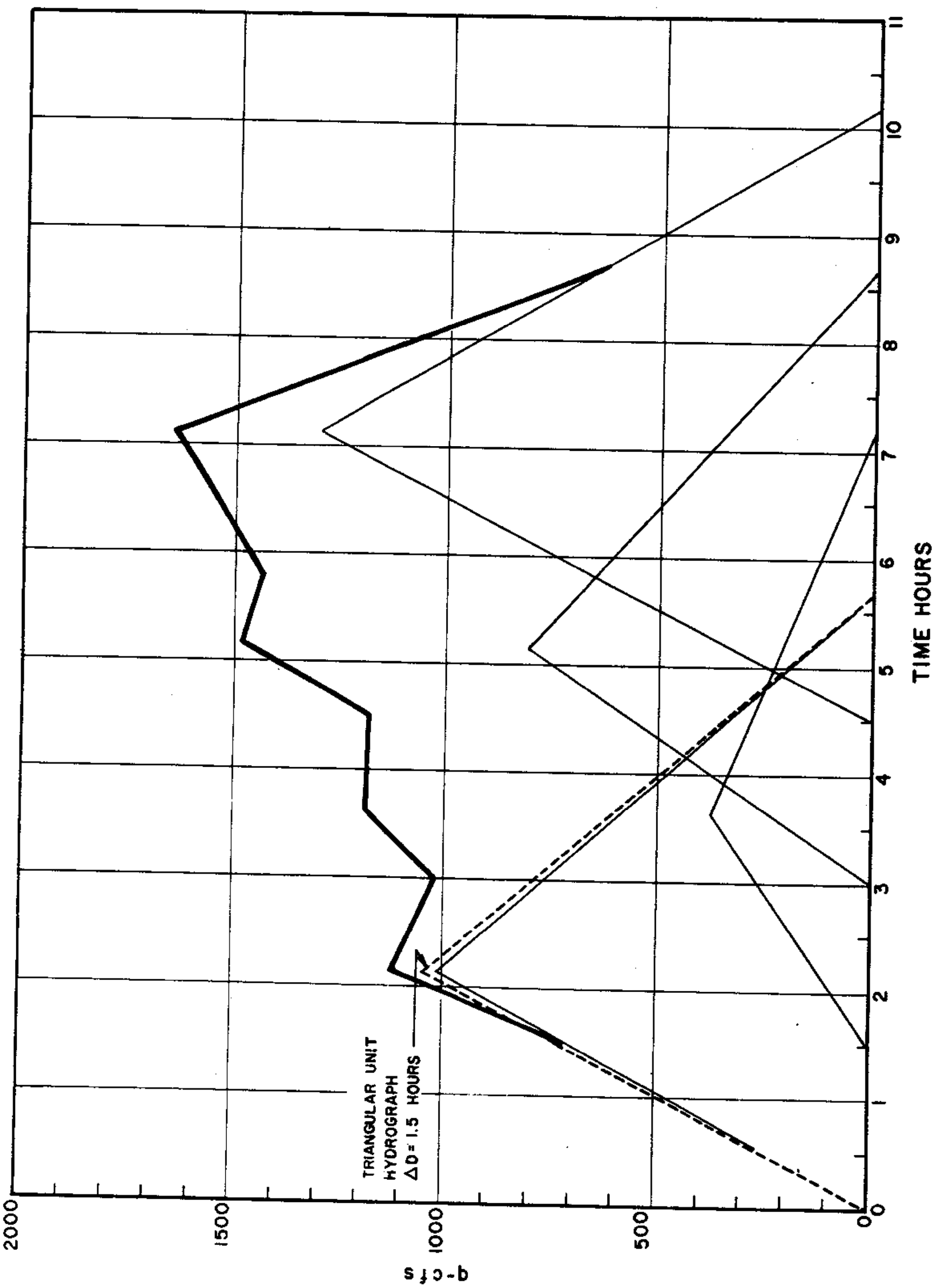

Figure 16.8 Composite flood hydrograph from example 3 showing effect when ΔD is too large

Examples 1 and 2 show that there is very little difference in the flood hydrograph developed using either a curvilinear unit hydrograph or a triangular unit hydrograph providing the unit of time (ΔD) is approximately 0.2 the time to peak of the unit hydrograph. This is the time defined by Mitchell, 1948, as the optimum time of a unit storm. Example 3 shows the effect of increasing the time increment to 1.5 hours which is approximately equal to the time to peak of the unit hydrograph when the optimum time increment is used.

Peak Discharge Determination

In using the triangular unit hydrograph to develop composite flood hydrographs, the peak of each triangular unit hydrograph is determined by multiplying the peak for one inch of runoff by the amount of runoff in each ΔD time. Assuming uniform runoff for an indefinite period of time and using ΔD as 0.2 of the time to peak of the unit hydrograph, figure 16.9 shows that 13 increments of runoff is the maximum number that will add to the peak discharge of the flood hydrograph. It also shows the percent of the peak of each incremental hydrograph that contributes to the peak of the composite flood hydrograph.

Table 16.7, column 2, shows a tabulation of these percentages in decimal form. This tabulation is used to compute the peak discharge and time to peak for any duration or pattern of rainfall.

Example 4

Compute the peak discharge and time to peak produced by the runoff from the rainfall shown in figure 16.4(b) and Table 16.6 for two locations on a homogeneous watershed. Given the following information:

Location 1 - Drainage Area, 2 square miles; T_c - 1.5 hours; CN-85.

Location 2 - Drainage Area, 20 square miles; T_c - 6 hours; CN-85; storm duration - 12 hours.

For Location 1:

Step 1. Compute the time increment ΔD.

From equation 16.12: $\Delta D = .133 \times 1.5 = .2$ hour

Step 2. Compute q_p the peak discharge for the unit hydrograph.

From equation 16.9:

$$q_p = \frac{484 \times 2 \times 1}{\frac{.2}{2} + .9} = 968 \text{ cfs}$$

Step 3. Knowing that 13 ΔD's is the maximum number of runoff increments that will contribute to the peak of the flood hydrograph, compute the maximum length of excess rainfall or runoff that will

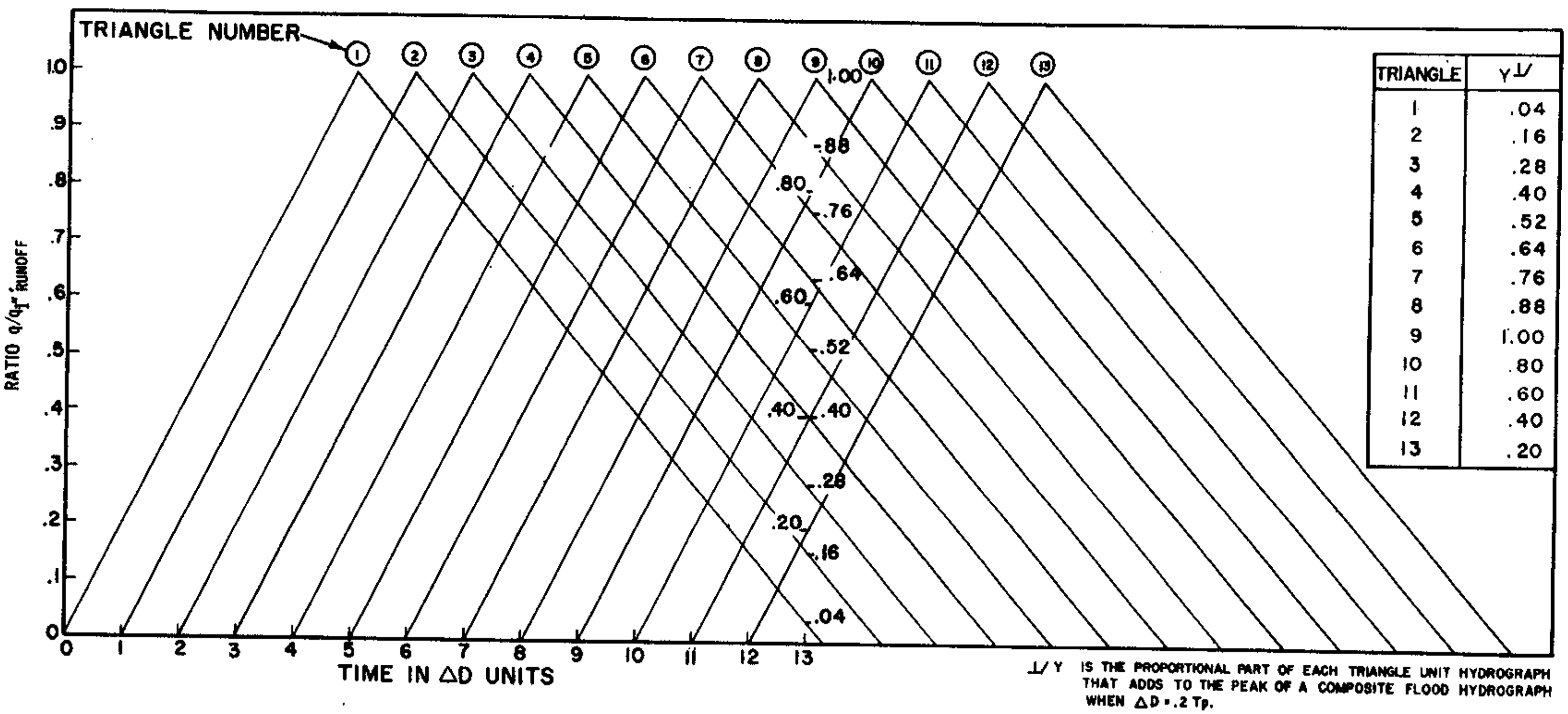

Figure 16.9 Part of triangular unit hydrograph that contributes to the peak when $\Delta D = 0.2\ T_p$

Table 16.6 Rainfall tabulated in 0.2 hour increments from plot of Rain Gage Chart, Figure 16.4(b)

Time (hours) (1)	Accum. Rainfall (in.) (2)	Accum.1/ Runoff (in.) (3)	ΔQ (in.) (4)
0	0	0	0
.2	.02	0	0
.4	.05	0	0
.6	.08	0	0
.8	.13	0	0
1.0	.20	0	0
1.2	.27	0	0
1.4	.36	0	0
1.6	.48	0	0
1.8	.60	.03	.03
2.0	.80	.09	.06
2.2	.95	.15	.06
2.4	1.18	.27	.12
2.6	1.45	.42	.15
2.8	1.68	.58	.16
3.0	2.00	.80	.22
3.2	2.22	.96	.16
3.4	2.42	1.12	.16
3.6	2.62	1.27	.15
3.8	2.82	1.43	.16
4.0	3.00	1.59	.16
4.2	3.10	1.68	.09
4.4	3.18	1.74	.06
4.6	3.20	1.76	.02
4.8	3.20	1.76	.00
5.0	3.21	1.77	.01
5.2	3.23	1.79	.02
5.4	3.38	1.91	.12
5.6	3.60	2.11	.20
5.8	3.83	2.31	.20
6.0	4.55	2.96	.65

Time (hours) (1)	Accum. Rainfall (in.) (2)	Accum.1/ Runoff (in.) (3)	ΔQ (in.) (4)
6.2	5.00	3.37	.41
6.4	5.35	3.70	.33
6.6	5.52	3.86	.16
6.8	5.68	4.01	.15
7.0	5.83	4.15	.14
7.2	6.00	4.31	.16
7.4	6.15	4.45	.14
7.6	6.30	4.59	.14
7.8	6.42	4.71	.12
8.0	6.54	4.82	.11
8.2	6.66	4.93	.11
8.4	6.80	5.06	.13
8.6	6.90	5.16	.10
8.8	7.02	5.28	.12
9.0	7.12	5.37	.09
9.2	7.21	5.46	.09
9.4	7.30	5.55	.09
9.6	7.40	5.64	.09
9.8	7.50	5.74	.10
10.0	7.60	5.84	.10
10.2	7.70	5.93	.09
10.4	7.80	6.03	.10
10.6	7.86	6.09	.06
10.8	7.90	6.12	.03
11.0	7.92	6.14	.02
11.2	7.94	6.16	.02
11.4	7.96	6.18	.02
11.6	7.98	6.20	.02
11.8	7.99	6.21	.01
12.0	8.00	6.22	.01

1/ Runoff computed using CN = 85 moisture condition II

Table 16.7 Peak discharge determined for example 4.

Triangle Number	Y1/	Location 1 ΔD = 0.2 hour: Time (hours)	Location 1: Runoff (inches)	Location 1: Col 2 x col 4	Location 2 ΔD = 0.8 hour Trial 1: Time (hours)	Trial 1: Runoff (inches)	Trial 1: Col 2 x col 7	Location 2 ΔD = 0.8 hours Trial 2: Time (hours)	Trial 2: Runoff (inches)	Trial 2: Col 2 x col 10
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	.04	4.2	.06	.0024						
2	.16	4.4	.02	.0032	.0	.0		.8	.0	
3	.28	4.6	.00	.0000	.8	.0		1.6	.27	.0756
4	.40	4.8	.01	.0040	1.6	.27	.1080	2.4	.69	.2760
5	.52	5.0	.02	.0104	2.4	.69	.3588	3.2	.63	.3276
6	.64	5.2	.12	.0768	3.2	.63	.4032	4.0	.17	.1088
7	.76	5.4	.20	.1520	4.0	.17	.1292	4.8	.35	.2660
8	.88	5.6	.20	.1760	4.8	.35	.3080	5.6	1.59	1.3992
9	1.00	5.82/	.65	.6500	5.62/	1.59	1.5900	6.42/	.61	.6100
10	.80	6.0	.41	.3280	6.4	.61	.4880	7.2	.51	.4080
11	.60	6.2	.33	.1980	7.2	.51	.3060	8.0	.46	.2760
12	.40	6.4	.16	.0640	8.0	.46	.1840	8.8	.36	.1440
13	.20	6.6	.15	.0300	8.8	.36	.0720	9.6	.39	.0780
				1.6948			3.9472			3.9692

1/ See figure 16.9 for definition of Y.

2/ The time to peak of the flood hydrograph is the time of beginning of incremental runoff opposite triangle number 9 plus the time to peak of the unit hydrograph.

contribute to the peak of a composite flood hydrograph at location 1 (0.2 x 13 = 2.6 hours). From table 16.6 note the maximum runoff for one ΔD (0.2 hour) is 0.65 inches, which occurs during the period from 5.8 to 6.0 hours from the beginning of rainfall.

Step 4. Tabulate the runoff in ΔD time increments each way from the maximum ΔD of runoff. There should be at least eight increments of runoff ahead of and four increments of runoff after the maximum increment as shown in table 16.7, column 4, where the ΔD increments of runoff are tabulated opposite the elapsed time after rainfall begins on the watershed.

Step 5. Multiply column 2 by column 4 and tabulate in column 5 of table 16.7.

Step 6. Compute the peak discharge and time to peak of the flood hydrograph at location 1 by multiplying the total of column 5 by the peak discharge of the unit hydrograph.

$$q_p = 1.695 \times 968 = 1640 \text{ cfs}$$

$$T_p = 5.8 + 1 = 6.8 \text{ hours (from beginning of rainfall)}$$

For Location 2:

Step 1. Compute the time increment, ΔD.
From equation 16.12, ΔD = .133 x 6 = .8 hour

Step 2. Compute q_p the peak discharge for the unit hydrograph. From equation 16.9:

$$q_p = \frac{484 \times 20 \times 1}{\frac{.8}{2} + 3.6} = 2420 \text{ cfs}$$

Step 3. Compute the maximum length of excess rainfall or runoff that adds to the peak of the composite flood hydrograph at location 2 (.8 x 13 = 10.4 hours). From table 16.6 the maximum runoff for one ΔD (.8 hour) is 1.59 inches and occurs during the period from 5.6 to 6.4 hours after the beginning of rainfall.

Step 4. Tabulate the runoff in ΔD time increments each way from the maximum ΔD of runoff. This tabulation is shown in table 16.7, column 7.

Step 5. Multiply column 2 by column 7 and tabulate in column 8.

Step 6. Compute the peak discharge and time to peak as shown in step 6 of example at location 1:

$$q_p = 3.947 \times 2420 = 9550 \text{ cfs}$$

$$T_p = 5.6 + 4.0 = 9.6 \text{ hours (from beginning of rainfall)}$$

Generally, the peak of the composite flood hydrograph can be computed by placing the largest increment of runoff opposite triangle number 9 as shown in table 16.7, column 1 and 4. However, if runoff is irregular, more than one computation may be required before determining the peak of the composite flood hydrograph. Trial 2 also is shown in table 16.7. In this case, the largest increment of runoff is placed opposite triangle number 8. Using the same procedure as in trial 1, the results are:

$$q_p = 3.969 \times 2420 = 9600 \text{ cfs}$$

$$T_p = 6.4 + 4.0 = 10.4 \text{ hours (from beginning of rainfall)}$$

Trial 2 shows that the peak discharge is greater when the largest increment of runoff is placed opposite triangle number 8. Other patterns of runoff may require several computations before the peak discharge is determined.

References

Mitchell, W. D., Unit Hydrographs in Illinois, State of Illinois, Division of Waterways, Springfield, Ill., 1948

Sherman, L. K., The Hydraulics of Surface Runoff, Civil Eng. 10:165-166, 1940